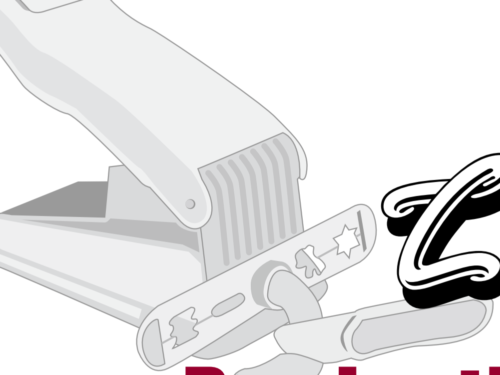

Licorice Production and Manufacturing: All-Sorts of Practical Applications for Statistics

JANE WATSON, JANE SKALICKY, NOLEINE FITZALLEN and **SUZIE WRIGHT** describe a rich task aimed at helping students from grade 1 to middle school appreciate the source of variation as they generate and examine their own data.

Among the practical applications of statistics is the collection of data from manufacturing processes. Often collected in the form of a time series, data collected from a series of measurements show the variation in those measurements, such as mass of a product manufactured. Limits are set for quality control and if these are exceeded then a decision is made about the process; perhaps it is halted and adjustments made to create a more uniform product over time. Can turning the primary (or middle school) classroom into a "food production line" promote learning outcomes that assist students in appreciating the essence of what the chance and data curriculum is about: variation? (Moore, 1990; Watson, 2007)

Konold and Harradine (2009) claim that although many teachers appreciate the need to provide students with real data, those data are often presented to students "cold," with the assumption that students can quickly gain a familiarity with the context from which they came. The focus of classroom activities often then remains on the computation of statistics and the drawing of graphs as mandated by the curriculum. Students are not likely, however, to appreciate the factors that produced the variation in the data, if they are not very familiar with the process that produced the data. For most students this "story behind the data" is too often inaccessible. Konold and Harradine therefore

believe that having students experience the process that creates variable data provides a powerful foundation on which students can build an understanding of data and motivate statistical ways of interpreting those data. Particularly powerful are contexts that involve the repeated actioning of a process that generates objects. This type of process allows learners to experience the "creation of variation" and sets the stage for defining the measurements they want to collect and choosing the graphic representations of those data that reveal both the variability and the structure in those data. Although it might seem as if many students have thought little about the variability of such processes and expect uniformity in all measures, becoming involved in the process themselves could provide first-hand experience of the variability produced.

The opportunity to mimic a manufacturing process arose from the work of Harradine (personal communication, 1 May 2007) through an activity presented in a professional development session for teachers as part of the StatSmart project (Callingham, Watson & Donne, 2008). The activity mimics the manufacturing of a product in two ways: "by hand" and "by machine." An extension looks at a commercially manufactured product. The first part of this article describes how a version of the activity was carried out in Grade 1 and Grade 3 classrooms. The second part of the article details a professional development session carried out with 27 teachers to introduce them to the concept of variation in an environment suitable for upper primary and middle school students.

Many variations on the design of the activity are possible, depending on time and resources. As described here, the activity is based on "manufacturing" licorice sticks from play dough, hence a quantity of play dough must be purchased or made. Licorice is chosen because students are likely to identify what they produce with commercially-produced licorice sticks. Given a set of standards for length and diameter, groups create their own play dough licorice sticks by hand and measure them. The sticks are then weighed and their masses recorded, to be displayed on a whole class stacked dot plot[1]. Then a Fun Factory Play-doh extruder is used to create another group of sticks with the same dimensions, and again they are measured and recorded[2]. It is expected that the variation in the second set of sticks will be less than the first. As an extension, packets of commercial licorice sticks can be measured and compared with the others for consistency[3]. Figure 1 shows the materials used with the students and teachers discussed in this article. Further, the links to the topics of measurement and estimation in the mathematics curriculum are clear, as is the potential for links to social science and innovation.

Digital scales

Commercial licorice

Play-doh Fun Factory

Figure 1. Materials for licorice activities.

[1] For this activity we used the UT Series Digital Pocket Scales available from Aidley Co., PO Box152, St Clair, NSW 2759.

[2] The Play-Doh Fun Factory is available from Hasbro Australia Ltd., 570 Blaxland Road, Eastwood, NSW 2122 (and found in most toy stores).

[3] An appropriate brand of licorice to use is the Ricci Traditional Soft Licorice (300 g) packet (available in most supermarkets).

Manufacturing in Grades 1 and 3

As part of activities for National Mathematics Day, students in two classes, Grades 1 and 3 in different schools, were introduced to the idea of variation with a discussion of the difference between home-made biscuits and those bought in the supermarket. The children were shown some of each and asked to describe the attributes of each that could be seen to vary. The consistency of shape and size of the supermarket biscuits was noted and students were asked to suggest reasons why these biscuits might be more similar than those made at home. Students were then asked, "If you bought a packet of licorice sticks would you expect them all to be the same?" At this point students were shown laminated colour photos of the production of licorice at the Licorice Factory in Junee, NSW. Three of these are produced in Figure 2. Further questions were then asked of the students: "What if the licorice sticks were hand-made rather than by a machine? What difference would we expect?"

Figure 2. Production of licorice at Junee Licorice Factory (www.greengroveorganics.com).

Students were shown the Fun Factory equipment and invited to try their skills at making play dough licorice sticks by hand themselves and with the Fun Factory machine. Students were asked which way would be able to create more consistent products and how they would test this. The commercial factory made 8 cm sticks, so it was decided to make a similar length by hand and with the Fun Factory. It was hence necessary for the children to measure lengths carefully with a ruler. It was decided that rolling the play dough to a diameter of approximately 1 cm would be appropriate. Measuring the mass of the sticks in grams then produced the attribute to be used to show and compare the consistency of the two manufacturing processes.

Students were divided into groups and each student made three licorice sticks by hand and three with the Fun Factory. In using the scales, the Grade 1 students recorded the number of grams shown on the electronic scales to the left of the decimal point. For the Grade 3 students, there was a discussion of decimals and they recorded their data to one decimal place (e.g., after examples showing that 7.3 g was bigger than 7.1 g). This was supported by a number line activity using rope and pegs to revise and consolidate place value understanding to one decimal place. Data for the two methods were recorded on sticky notes.

Figure 3 shows the students working in the Grade 1 class. Class discussion then took place on how to record the data collected and the minimum and maximum values were reported in order to decide the extreme values on the axes for each of the two stacked dot plots. In the Grade 1 class the data were recorded by the teacher (the third author) on the whiteboard as shown in Figure 4. The summing up of the activity took place in a class group, also shown in Figure 4. Student discussion of the appearance of the plots included describing the plots of the hand-made masses as a "hill," a "landscape," and "all kinds of shapes next to each other." For the plot of the factory-made masses, the responses from students included: a "tall building," "There are more in the middle," and "They are more the same." Although impressed with the consistency of the masses from the Fun Factory licorice sticks, many of

the students were convinced that making the sticks by hand was best, “because you can try harder to make a better shape.” Some also thought it would be an advantage because they “might” get more big ones. When the teacher returned to the class for National Mathematics Day the following year, the students remembered the licorice activity and could describe the outcomes and shapes of the plots.

Figure 3. Making licorice sticks in the Grade 1 class.

Figure 4. Outcomes and discussion of activity in the Grade 1 class.

Figure 5. Making licorice sticks in the Grade 3 class.

Figure 5 shows the process of making the licorice sticks by hand and with the Fun Factory machine in the Grade 3 class. In this class the students recorded their mass measurements on sticky notes, one colour for hand-made and another for factory-made. They then put the labels on two separate whiteboards themselves, making the decisions on the "axis" labels. The two plots produced are shown in Figure 6.

Figure 6. Plots for the hand-made and Fun-Factory-made licorice sticks in the Grade 3 class.

The whole class discussion (led by the second author) focussed on the following questions:

- Is one group more spread out than the other?
- What is the difference in spread between the two groups?
- Is there a difference in the middle of the plots?
- Which is more consistent?
- Would we expect the same plot if we did our sample of licorice sticks again?
- Would we be better at producing licorice sticks?
- What would "better" mean?

After the discussion students were given the sheet shown in Figure 7 to summarise what they had learned from the activity.

PLAY-DOH FACTORIES

Today we made hand-made licorice sticks and factory-made licorice sticks. We weighed all of our licorice sticks to see how good we are at making them all the same.

We investigated the questions:

How good are we at making licorice sticks all the same?

When we hand-made licorice sticks how different were they?

When we made licorice sticks using the play-doh factory how different were they?

Write some sentences about what you found out:

Draw a picture that shows what you found out:

Figure 7. Play-Doh Factories worksheet for students.

Following are examples of what the students wrote about their findings:

- The licorice sticks were different because the factory sticks were lighter and the handmade weren't as perfect because they were longer and fatter. But the factory wons (sic) were short and thinner. The factory had a hole in it to make them all the same. (S1)
- The hand-made and the machine-made ones do not weigh the same. The hand-made ones were heavier than the machine ones. That licorice is 8 cm long. The machine ones came out the same width and the hand-made ones were heavier because we might have put extra play-doh in it. (S2)
- The factory made was just about exactly the same but the hand made was completely different. Because the factory made does the same thing every time. But the hand made doesn't do the same thing every time. (S3)
- Even though the factory made licorice sticks are not the same size, hand made ones have got a larger range. (S4)
- The hand made licorice strips were like 10 to 20 grams apart while the factory made ones were only 1 to 2 grams apart. The factory made ones came at the same size and the hand made ones came out way different. (S5)

Four of the drawings students made to show what they found are produced in Figure 8.

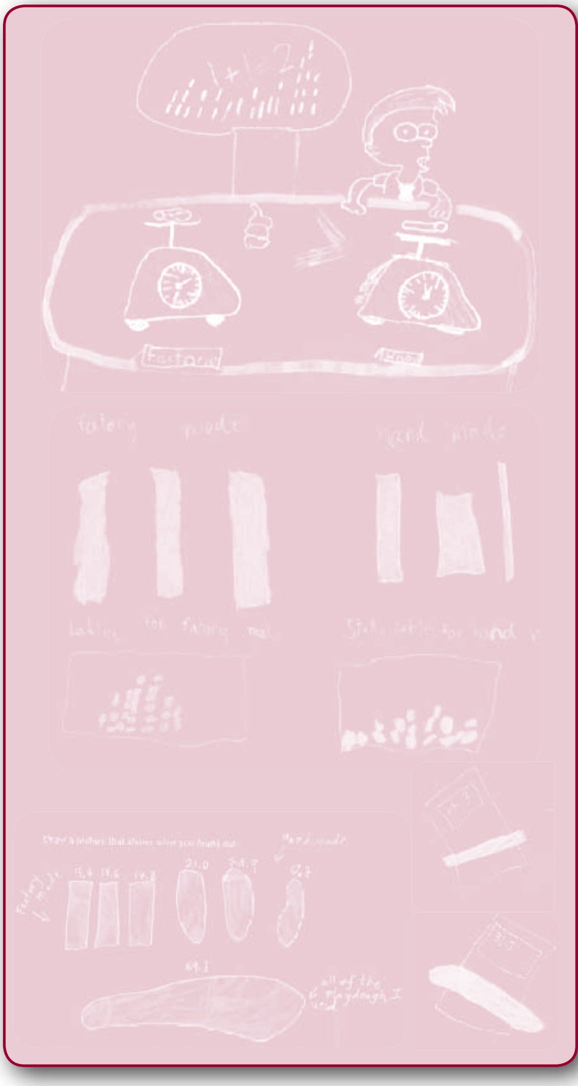

Figure 8. Pictures of what four Grade 3 students found out from the licorice activity.

The activities in both classrooms provoked a high level of student interest and concentration. Care in measurement, without very much prompting, was evident in both classes. In Grade 1 the measurement skills, especially with rulers, were well beyond the level of curriculum expectations. The hands-on nature of the activity was popular with both classes and the comparison of the two methods of production created intense interest. The idea of "better" being more uniform with less variation was consolidated more confidently in the Grade 3 class than the Grade 1 class, where "better" was more likely to be associated with the creative act of making the licorice sticks by hand and making larger sticks, than with uniformity of measurement. The students in Grade 1 did, however, appreciate the greater variation in the licorice sticks made by hand.

Manufacturing with middle school teachers

The licorice activity as implemented with middle school teachers was structured in a more complex fashion than the activities with Grades 1 and 3. It was intended to model the activity with a range of possibilities that could be adapted for middle school students of varying abilities.

The data used in this article were collected from 27 teachers participating in a chance and data professional learning session of about two hours (organised by the first and fourth authors). The initial discussion introduced the idea of a manufacturing process and the need for uniformity in products produced. The teachers were aware of quality control regimes and the measurement of variation from an ideal size, shape, mass, or quantity. It was agreed that this was a good context in which to introduce variation to upper primary and middle school students and to challenge them to "manufacture" a product consistently. Using the context of manufacturing play dough licorice sticks 1 cm in diameter and 8 cm long, similar to those that could be made by the Fun Factory extruder, the hypothesis for the activity was: "We can make licorice sticks as consistently as the Fun Factory machine." The availability of commercially manufactured licorice sticks led to the further question of the variation in the genuinely manufactured product as well. Hence the investigation sought to explore the variation in the products of the three processes with the expectation that the observed variation would decrease from the hand-made to the Fun Factory manufactured to the commercially manufactured licorice sticks.

After the initial introduction took place, teachers were split into nine groups of three teachers and each group took part in each of the three activities on a rotational basis. Although all teachers did not complete the three activities in the same order, they experienced all three. The groups created between 20 and 47 licorice sticks by hand, based on their estimates of the diameter and length, weighing each stick and recording the mass on a sticky note. A similar process was used with the Fun Factory machine, the groups producing between 25 and 38 licorice sticks. These were then added to large graphs on whiteboards; different coloured sticky notes used for each group. For the commercially manufactured licorice sticks, both the length and the mass were measured. As this was happening, one of the leaders was entering the data for each group into a TinkerPlots file (Konold & Miller, 2005). Teachers in this session had TinkerPlots in their schools and it was intended that they would have two methods of displaying the data with their students: with sticky notes on whiteboards and with TinkerPlots graphs. Some of the advantages of using TinkerPlots are shown in the figures in this section. [For more information on TinkerPlots and activities employing the software, see: Watson (2008); Watson, Fitzallen, Wilson & Creed (2008); and Watson & Wright (2008).]

Discussion of the representations first focussed on the three whiteboards, looking at within group and between group variation (Harradine, 2005). Data for different groups could be distinguished by the different coloured sticky notes and the consistency of one group for the hand-made sticks stood out immediately, with that group being accused of cheating and weighing their play dough before creating their licorice sticks.

Later, similar trends were noted in the TinkerPlots graphs and these are reproduced in the figures here. Given the overall variation in the graph of the hand-made licorice sticks, one group (Group F) again was quickly seen as an "outlier" in terms of small variation in its production. As well there was discussion about the different type of outlier production by Group G, whose sticks were consistently of greater mass. This group claimed to have been given more dense play dough![4] Seeing the representations in TinkerPlots (Figure 9) and comparing the variation across groups provided strong evidence of the claim the teachers were making. The features in TinkerPlots that helped to describe the differences in the groups' licorice-making skills included the means for groups (Δ) and hats, which show the approximate middle 50%, and lower and upper 25% of the data sets. Teachers agreed that this type of representation would create interest in their classes as groups compared their manufacturing skills.

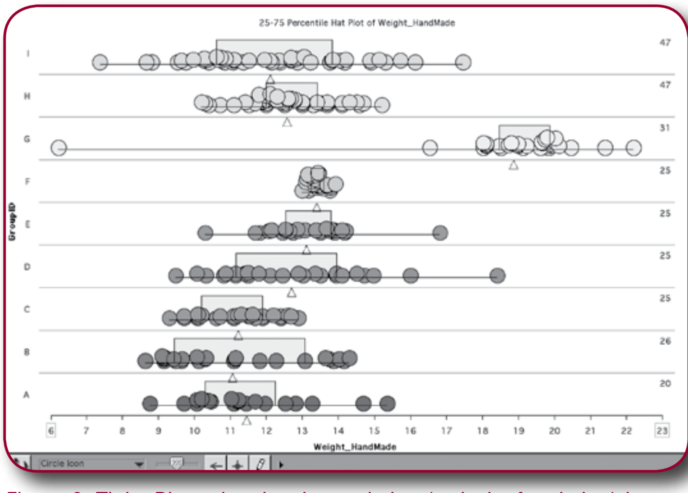

Figure 9. TinkerPlots plot showing variation (or lack of variation) in masses of hand-made licorice sticks made by different groups.

Initially, and somewhat surprisingly for the teachers, the variation in the Fun Factory licorice sticks created by individual groups did not appear that much less than that of the hand-made ones using the plots on the whiteboard. When considered in TinkerPlots, however, with the end points of the axes made the same, the lesser variation in the masses became apparent. This is shown in Figure 10.

[4] This claim was in fact true because most groups had "home-made" play dough, whereas one group did have the play dough that came with the Fun Factory kit.

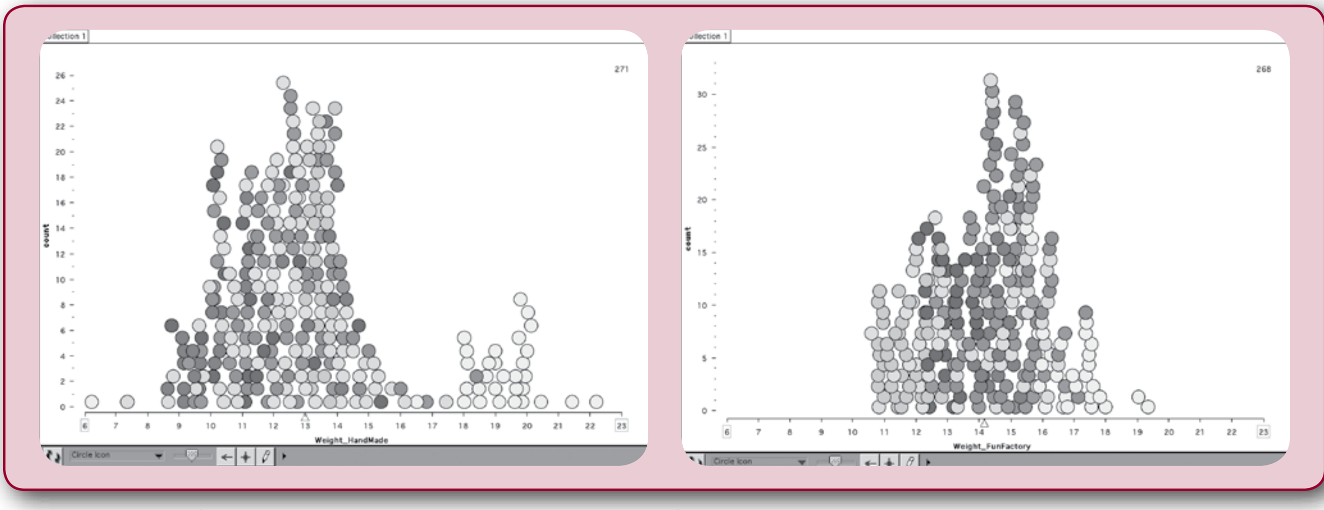

Figure 10. The data for the hand-made and Fun Factory licorice sticks for all groups displayed in TinkerPlots using the same axis endpoints.

When considering the Fun Factory data by group as shown in Figure 11, the variation for Group F is no longer unusually small compared say to Groups A and B, so it was decided that perhaps they did not cheat on this part of the task. The greater average mass for Group G was seen to support their claim of denser play dough. The smaller variation in the groups is also seen again by comparing Figure 11 with Figure 9. The ability to produce such graphs efficiently for groups within a class is a motivational aspect for collaborative group work.

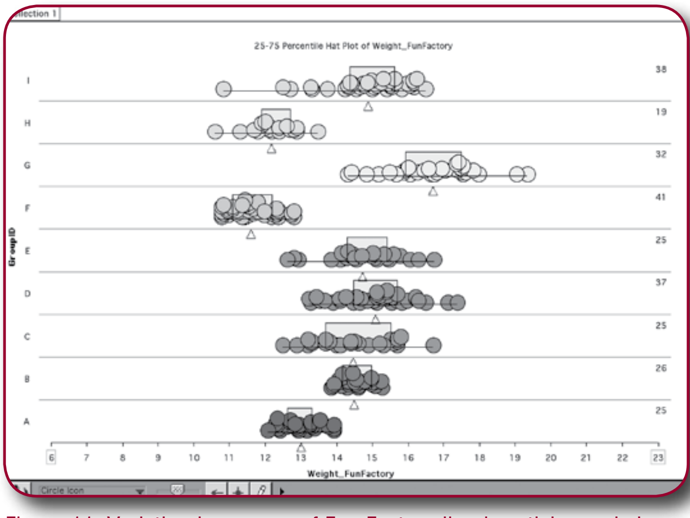

Figure 11. Variation in masses of Fun Factory licorice sticks made by different groups.

For the commercially produced licorice sticks, each group was given a new unopened packet and carefully measured the mass of each stick in the packet. There was considerable discussion about the masses of these licorice sticks. One packet had a tiny piece of licorice weighing 2 g and this was recorded but later eliminated as an outlier. Another packet (measured by Group G) had sticks that were individually lighter than the others and the teachers suggested that it must have come from a different production run at the factory (see Figure 12). Eliminating Group G's packet and using the same endpoints on the axis as were used in Figure 9 demonstrated well the smaller range and variation in the masses of the commercial product (see Figure 13). The presence of Group G's packet, however, provided another stimulus for discussion. First was the question of whether the total weight claimed on the outside of the packet had been reached. When the number of licorice sticks in Group G's packet was counted, however, there were 30 in it, compared to 23 to 26 in all other packets. Several groups then calculated the total mass of the sticks in their packets to see if it reached 300 g. Figure 14, with the labelled mean values and the number of sticks in each packet, provides a quick way of checking this matter. It is also a good way of reinforcing understanding of the formula for calculating the mean for middle school students, by working "backward" to find the total mass of the packets' contents. Teachers found it interesting that the range of total grams in the packets was 309 g to 331 g.

The teachers were hence satisfied that one quality control measure (total mass) could compensate for another (lighter mass per individual stick).

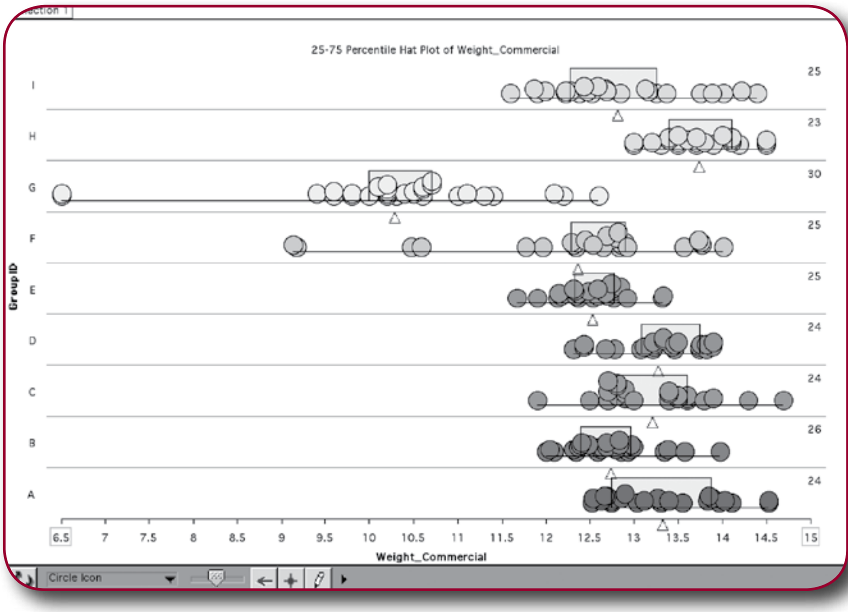

Figure 12. Masses of licorice sticks from 9 commercially produced packets (labelled 300g).

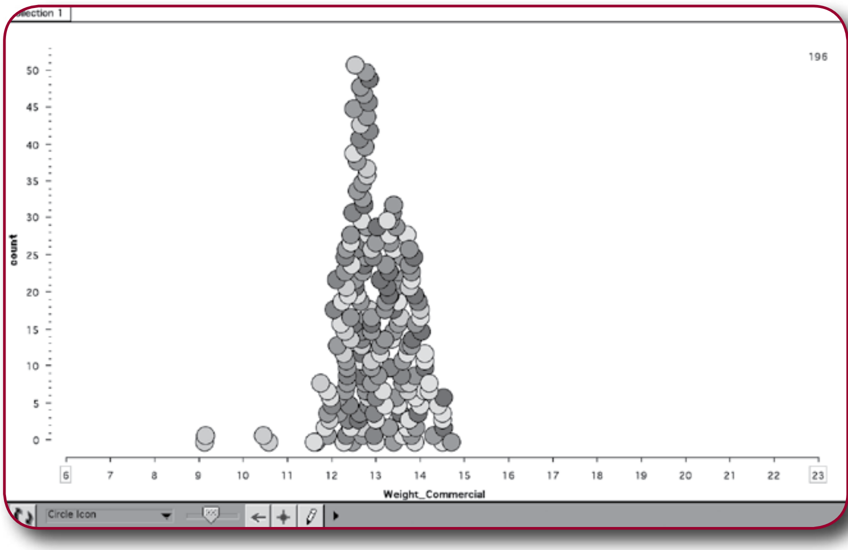

Figure 13. Combined data set from 8 commercially produced packets of licorice, assumed from the same production run.

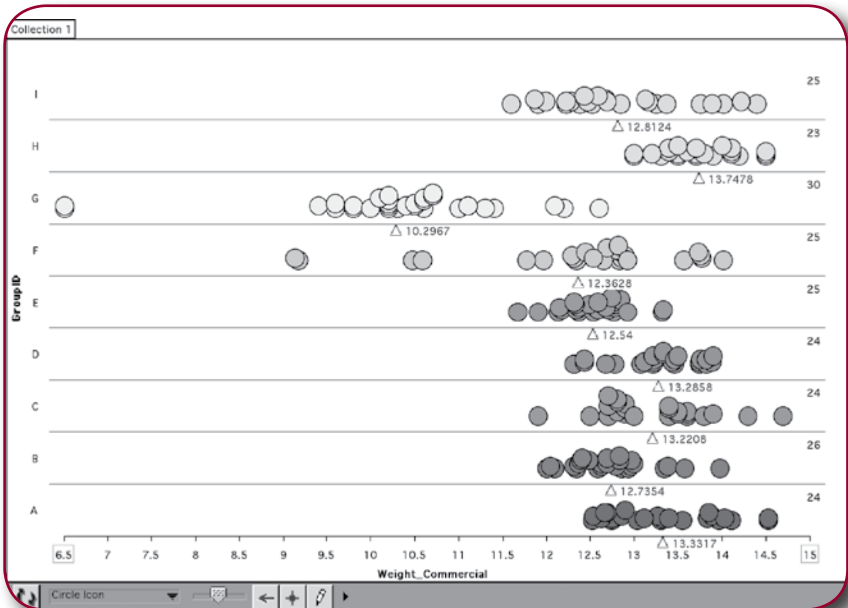

Figure 14. Masses for 9 packets of commercially produced licorice sticks labelled with mean values and sample size (n) to facilitate calculation of total packet mass.

Conclusion

These activities provide a wide range of experiences suitable for students from Grade 1 to the middle years. They are especially useful in exposing students to the concept of variation, which traditionally has been left to later years. They also illustrate the power of the sequence of experience suggested by Harradine and Konold (2009) and shown in Figure 15. Although it requires planning and the collection of materials, the effort is also well worthwhile in terms of exposing students to links among various parts of the mathematics curriculum and to an authentic application in the real world.[5] The use of TinkerPlots as a technology dimension demonstrates the importance of statistical software both for educational purposes (Fitzallen, 2007) as shown here and for quality control purposes as related software is used in the manufacturing industry. Besides all of the learning outcomes there may be the added reward of licorice to share at the end of the lesson!

Figure 15. Experience of variation to develop understanding.

Acknowledgements

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[5] Have you ever wondered if a packet you picked up in the supermarket contained the mass it claimed? Have you ever taken it to the produce section to check it out? One of our teachers has and hence has wonderful stories to accompany the activity in her classroom.

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